A transforming method from non-homogeneous evaluation value into IVIFNs

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Abstract: In this study, the author proposes a systematic approach to evaluate alternatives with heterogeneous attribute. This method is based on the transforming method proposed by Wan et al. (2016). For decision making problems in construction field, most problems belong to the type of non-homogenous ones as it contains both objective and subjective attribute. This method can hopefully provide hints to optimal selection in construction of building members, equipment and techniques.

1. Introduction

There are a large variety of decision-making methods such as TOPSIS, AHP, ANP, etc. However, most of these method are not quite suitable for solving practical problems, as real life situations usually involve both subjective and objective factors. Most of study quantify subjective evaluation values by crisp numbers, thus all the evaluation values are presented in one form.

To deal with non-homogenous group decision making problem, Wang and Liu (2013) developed an extended LINMAP method for multi-criteria group decision making under interval-valued intuitionistic fuzzy environment. Zhang and Xu (2015) adopted a deviation modeling approach to deal with non-homogeneous decision making problem. Li et al. (2010) proposed a systematic approach to solve this kind of problem. Compared with previous methods, the method proposed by Wan et al. (2016) can better cope with vagueness of evaluation information by transforming different data formats into unified form of interval-valued intuitionistic fuzzy numbers (IVIFNS).

2. A novel method of transforming non-homogeneous information into IVIFNs

In this part, we introduce four steps involved in this transforming process. By following this process, the decision-making evaluation values are unified into one format of IVIFNs that can better cope with imprecision in complex decision making environment.

Step 1. Transform the initial heterogeneous evaluation matrix $X^k = (x_{ij}^k)_{m\times n}$ to the normalized evaluation matrix by Eq. (1):

$$\hat{X}^k = (\hat{x}_{ij}^k)_{m\times n}, \tag{1}$$

where $\hat{x}_{ij}^k$ can be calculated by Eq. (2):

$$\hat{x}_{ij}^k = \begin{cases} a_{ij}^k / a_j^+, & if\ j \in N_1^b \\ a_j^+ / a_{ij}^k, & if\ j \in N_1^c \\ [a_{ij}^k / b_j^+, b_{ij}^k / b_j^+], & if\ j \in N_2^b \\ [a_j^+ / b_{ij}^k, a_j^+ / a_{ij}^k], & if\ j \in N_2^c \\ (a_{ij}^k / c_j^+, b_{ij}^k / c_j^+, c_{ij}^k / c_j^+), & if\ j \in N_3^b \\ (a_j^+ / c_{ij}^k, a_j^+ / b_{ij}^k, a_j^+ / a_{ij}^k), & if\ j \in N_3^c \end{cases} \tag{2}$$

where N_i^c and N_i^b represent the subscript sets of cost and benefit criteria. Besides, $i = 1, 2, 3$ represent evaluation values in the formats of real numbers , interval numbers and triangle fuzzy numbers respectively. Besides, $\hat{x}_j^+$ is the largest and smallest of j^{th} benefit and cost criteria respectively. For benefit criteria, if $j \in N_1$, $x_{ij}^k = a_{ij}^k$, then $a_j^+ = \max_{1 \le i \le m} a_{ij}^k$; if $j \in N_2$, $x_{ij}^k = [a_{ij}^k, b_{ij}^k]$, then $b_j^+ = \max_{1 \le i \le m} b_{ij}^k$; if $j \in N_3$, $x_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$, then $c_j^+ = \max_{1 \le i \le m} c_{ij}^k$. For cost criteria, if $j \in N_1$, $x_{ij}^k = a_{ij}^k$, then $a_j^+ = \min_{1 \le i \le m} a_{ij}^k$; if $j \in N_2$, $x_{ij}^k = [a_{ij}^k, b_{ij}^k]$, then $a_j^+ = \min_{1 \le i \le m} a_{ij}^k$; if $j \in N_3$, $x_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$, then $a_j^+ = \min_{1 \le i \le m} a_{ij}^k$. Afterwards, $\hat{x}_{ij}^k$ are normalized into values from 0 to 1 and transformed into benefit criteria.

Step 2. Determine Qsd (ξ_{ij}^k), Qdd (ζ_{ij}^k) and Qud (η_{ij}^k) of the element x_{ij}^k in A_{kj}.

(1) If $j \in N_1$, then we can get $A_{kj} = (x_{1j}^k, x_{2j}^k, \ldots, x_{mj}^k)^T$, which is in the form of real numbers (i.e., $x_{ij}^k = a_{ij}^k, i = 1, 2, \ldots, m$), and $x_j^+ = A_j^{\max}$. Then we can get ξ_{ij}^k, ζ_{ij}^k and η_{ij}^k of real number x_{ij}^k by Eqs. (3-5):

$$\xi_{ij}^k = \frac{a_{ij}^k}{x_j^+}, \tag{3}$$

$$\zeta_{ij}^k = 1 - \xi_{ij}^k, \tag{4}$$

$$\eta_{ij}^k = \begin{cases} \dfrac{x_j^+ - a_{ij}^k}{\frac{1}{2} x_j^+}, & \text{if } a_{ij}^k > \frac{1}{2} x_j^+ \\ \dfrac{a_{ij}^k}{\frac{1}{2} x_j^+}, & \text{if } a_{ij}^k < \frac{1}{2} x_j^+ \\ 1, & \text{if } a_{ij}^k = \frac{1}{2} x_j^+ \end{cases} \tag{5}$$

(2) If $j \in N_2$, then we can get $A_{kj} = (x_{1j}^k, x_{2j}^k, \ldots, x_{mj}^k)^T$, which is in the form of interval numbers (i.e., $x_{ij}^k = [a_{ij}^k, b_{ij}^k], i = 1, 2, \ldots, m$). Let $x_j^+ = A_j^{\max}$, then we can get ξ_{ij}^k, ζ_{ij}^k and η_{ij}^k of the interval number x_{ij}^k by Eqs. (6-8):

$$\xi_{ij}^k = \frac{a_{ij}^k + b_{ij}^k}{2 x_j^+}, \tag{6}$$

$$\zeta_{ij}^k = 1 - \xi_{ij}^k, \tag{7}$$

$$\eta_{ij}^k = \begin{cases} \dfrac{2x_j^+ - a_{ij}^k - b_{ij}^k}{\frac{3}{2} x_j^+ + \left| \frac{1}{2} x_j^+ - a_{ij}^k \right| - a_{ij}^k}, & \text{if } a_{ij}^k + b_{ij}^k > x_j^+ \\ \dfrac{a_{ij}^k + b_{ij}^k}{b_{ij}^k + \frac{1}{2} x_j^+ + \left| b_{ij}^k - \frac{1}{2} x_j^+ \right|}, & \text{if } a_{ij}^k + b_{ij}^k < x_j^+ \\ 1, & \text{if } a_{ij}^k + b_{ij}^k = x_j^+ \end{cases} \tag{8}$$

(3) If $j \in N_3$, then we can get $A_{kj} = (x_{1j}^k, x_{2j}^k, ..., x_{mj}^k)^T$, which is in the form of TFNs (i.e., $x_{ij}^k = [a_{ij}^k, b_{ij}^k, c_{ij}^k], i = 1,2,...,m$). Let $x_j^+ = A_j^{\max}$, then we can obtain ξ_{ij}^k, ζ_{ij}^k and η_{ij}^k of TFN x_{ij}^k by Eqs. (9-11):

$$\xi_{ij}^k = \frac{a_{ij}^k + 2b_{ij}^k + c_{ij}^k}{4x_j^+}, \tag{9}$$

$$\zeta_{ij}^k = 1 - \xi_{ij}^k, \tag{10}$$

$$\eta_{ij}^k = \begin{cases} \dfrac{4x_j^+ - a_{ij}^k - 2b_{ij}^k - c_{ij}^k}{\left|a_{ij}^k - \frac{1}{2}x_j^+\right| + 2\left|b_{ij}^k - \frac{1}{2}x_j^+\right| + \frac{7}{2}x_j^+ - a_{ij}^k - 2b_{ij}^k}, & \text{if } a_{ij}^k + 2b_{ij}^k + c_{ij}^k > 2x_j^+ \\ \dfrac{a_{ij}^k + 2b_{ij}^k + c_{ij}^k}{2\left|b_{ij}^k - \frac{1}{2}x_j^+\right| + \left|c_{ij}^k - \frac{1}{2}x_j^+\right| + 2b_{ij}^k + c_{ij}^k + \frac{1}{2}x_j^+}, & \text{if } a_{ij}^k + 2b_{ij}^k + c_{ij}^k < 2x_j^+ \\ 1, & \text{if } a_{ij}^k + 2b_{ij}^k + c_{ij}^k = 2x_j^+ \end{cases} \tag{11}$$

Step 3. Determine the quasi-IVIFN.

To derive quasi-IVIFNs, we can firstly obtain Qsi ($\tilde{\xi}_{kj}$),Qdi ($\tilde{\zeta}_{kj}$) and Qui ($\tilde{\eta}_{kj}$) by Eqs. (12-14):

$$\tilde{\xi}_{kj} = [\xi_{kj}^l, \xi_{kj}^h] = [\max(m(\xi_{kj}) - d(\xi_{kj}), 0), m(\xi_{kj}) + d(\xi_{kj})], \tag{12}$$

$$\tilde{\zeta}_{kj} = [\zeta_{kj}^l, \zeta_{kj}^h] = [\max(m(\zeta_{kj}) - d(\zeta_{kj}), 0), m(\zeta_{kj}) + d(\zeta_{kj})], \tag{13}$$

$$\tilde{\eta}_{kj} = [\eta_{kj}^l, \eta_{kj}^h] = [\max(m(\eta_{kj}) - d(\eta_{kj}), 0), m(\eta_{kj}) + d(\eta_{kj})], \tag{14}$$

and $m(\xi_{kj}) = \frac{1}{m}\sum_{i=1}^m \xi_{ij}^k$, $m(\zeta_{kj}) = 1 - m(\xi_{kj})$, $m(\eta_{kj}) = \frac{1}{m}\sum_{i=1}^m \eta_{ij}^k$, $d(\xi_{kj}) = \sqrt{\frac{1}{m-1}\sum_{i=1}^m (\xi_{ij}^k - m(\xi_{kj}))^2}$, $d(\zeta_{kj}) = \sqrt{\frac{1}{m-1}\sum_{i=1}^m (\zeta_{ij}^k - m(\zeta_{kj}))^2}$, $d(\eta_{kj}) = \sqrt{\frac{1}{m-1}\sum_{i=1}^m (\eta_{ij}^k - m(\eta_{kj}))^2}$, which are the mean values and standard deviations of ξ_{ij}^k, ζ_{ij}^k and η_{ij}^k respectively. Then an ordered pair named quasi-IVIFN $\alpha_{kj}^q = < [\xi_{kj}^l, \xi_{kj}^h], [\zeta_{kj}^l, \zeta_{kj}^h] >$ can be developed.

Step 4. Calculate IVIFN from quasi-IVIFN $\alpha_{kj}^q = < [\xi_{kj}^l, \xi_{kj}^h], [\zeta_{kj}^l, \zeta_{kj}^h] >$ by Eq. (15):

$$\alpha_{kj} = < [\mu_{kj}^l, \mu_{kj}^h], [v_{kj}^l, v_{kj}^h] > \tag{15}$$

where $\mu_{kj}^l = \xi_{kj}^l / \psi_{kj}$, $\mu_{kj}^h = \xi_{kj}^h / \psi_{kj}$, $v_{kj}^l = \zeta_{kj}^l / \psi_{kj}$, $v_{kj}^h = \zeta_{kj}^h / \psi_{kj}$ and $\psi_{kj} = \xi_{kj}^l + \xi_{kj}^h + \zeta_{kj}^l + \zeta_{kj}^h + \frac{1}{2}(\eta_{kj}^l + \eta_{kj}^h)$. The calculation process is demonstrated with the following Tables 1-3.

Table.1. Normalized comparison table

alternative	Experts	C_1	C_2	C_3
A_1	D_1	0.65	(0.25,0.50,0.75)	[0.80,1.00]
	D_2	0.65	(0.50,0.75,1.00)	[0.80,1.00]
	D_3	0.65	(0.25,0.50,0.75)	[0.80,1.00]
	D_4	0.65	(0.25,0.50,0.75)	[0.80,1.00]

Table.2. Qsd (ξ_{ij}^{k}), Qdd (ζ_{ij}^{k}) and Qud (η_{ij}^{k}) of each evaluation value

Alternative	Experts	C_1	C_2	C_3
A_1	D_1	0.65,0.35,0.70	0.50,0.50,1.00	0.90,0.10,0.20
	D_2	0.65,0.35,0.70	0.75,0.25,0.50	0.90,0.10,0.20
	D_3	0.65,0.35,0.70	0.50,0.50,1.00	0.90,0.10,0.20
	D_4	0.65,0.35,0.70	0.50,0.50,1.00	0.90,0.10,0.20

Table.3. Qsi ($\tilde{\xi}_{kj}$), Qdi ($\tilde{\zeta}_{kj}$) and Qui ($\tilde{\eta}_{kj}$) and aggregated IVIFNs

Schemes		C_1	C_2	C_3
A_1	$\tilde{\xi}_{1j}$	[0.65,0.65]	[0.44,0.69]	[0.90,0.90]
	$\tilde{\zeta}_{1j}$	[0.35,0.35]	[0.31,0.56]	[0.10,0.10]
	$\tilde{\eta}_{1j}$	[0.70,0.70]	[0.63,1.13]	[0.20,0.20]
	IVIFNs	< 0.31,0.31], [0.13,0.13]>	< [0.15,0.24], [0.11,0.20]>	<[0.41,0.41], [0.05,0.05]>

3. Conclusion

This method with the above-mentioned steps can transform different data formats into one unified form, thus facilitating subsequent data process in terms of optimal selection.

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